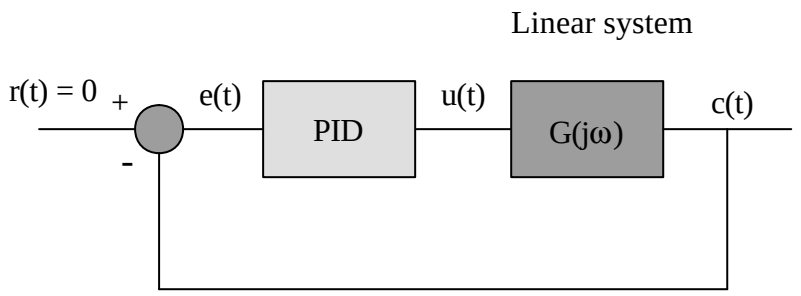
PID CONTROL TUNING

Basic problem

Determine PID control parameters by experiments.



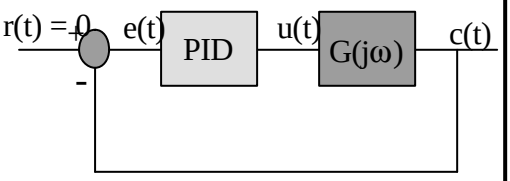
PID

$$u(t) = K_p e(t) + K_i \int_0^t e(a)\,da + K_d \frac{de}{dt}$$

$$= K_p \left(e(t) + T_i \int_0^t e(a)\,da + T_d \frac{de}{dt} \right)$$

$$U(s) = \left[K_p + K_i \frac{1}{s} + K_d s \right] E(s)$$

Practical problem with derivative control



PID

Ideal

$$U(s) = \left[K_p + K_i \frac{1}{s} + K_d s \right] E(s)$$

PID

Practical

$$\frac{U(s)}{E(s)} = K_p + K_i \frac{1}{s} + \frac{K_d s}{\frac{s}{b} + 1}$$

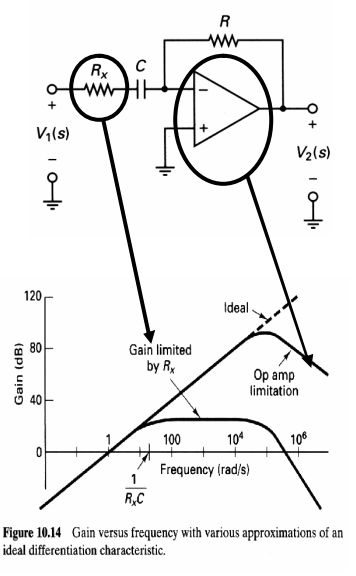
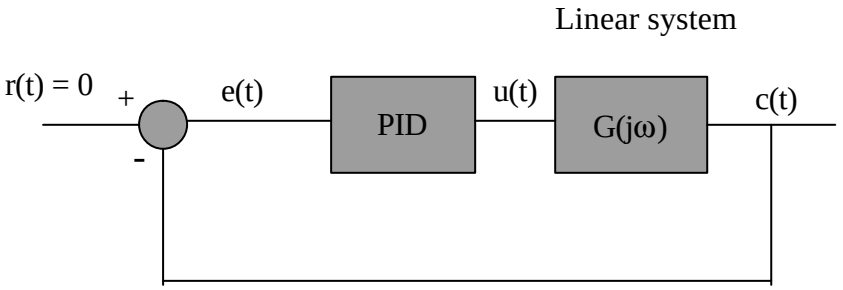


Figure 10.14 Gain versus frequency with various approximations of an ideal differentiation characteristic.

Tuning methods

- Open loop tuning
- Closed-loop tuning
- Relay tuning
- Optimization of control parameters
- Parameter estimation methods
- Rule-based methods
- Gain scheduling
- Adaptive control

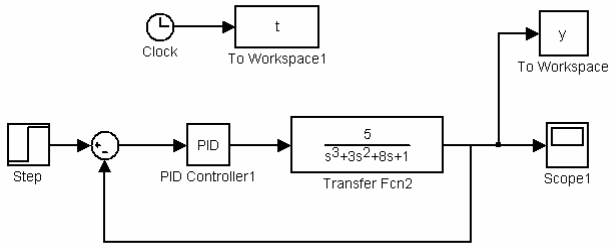
Closed-loop tuning



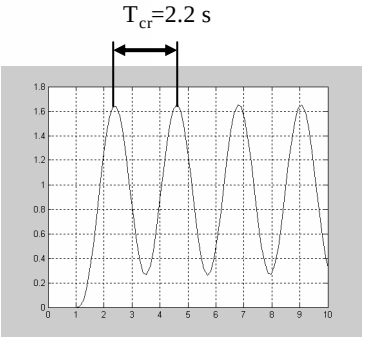
1. Set $T_i = \infty$ and $T_d = 0$.
2. Increase K_p until the system oscillates to obtain critical gain K_{cr} and critical period T_{cr} *(or frequency)*

$$u(t) = K_p\left(e(t) + T_i\int_0^t e(a)da + T_d\frac{de}{dt}\right)$$

Closed-loop Ziegler Nichols



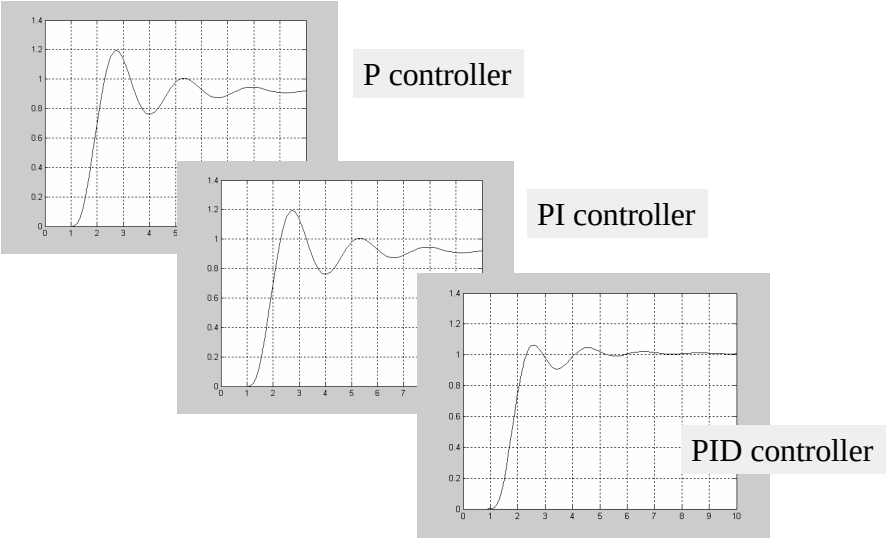
K_{cr}=4.6
T_{cr}=2.2 s



Closed-loop Ziegler Nichols

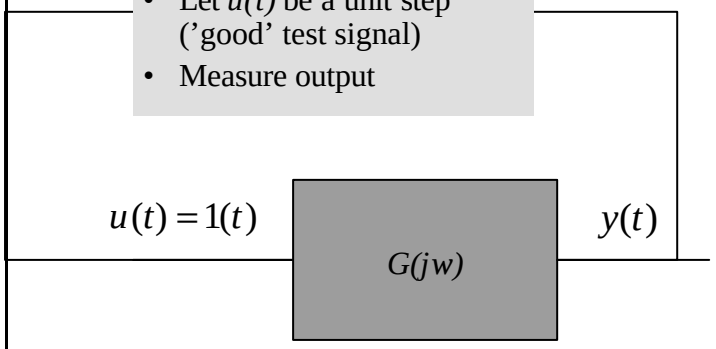
PID Controller parameters;			
Ziegler-Nichols frequency response method			
Control	K	Ti	Td
P	0.5Kcr=2.3		
PI	0.4Kcr=1.8	0.8Tcr=1.8	
PID	0.6Kcr=2.8	0.5Tcr=1.1	0.125Tcr=0.3

Closed-loop Ziegler Nichols

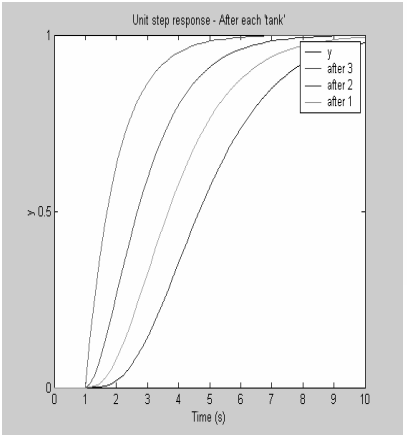


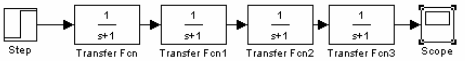
Open-loop tuning

- Open loop
- Identify plant dynamics
- Let *u(t)* be a unit step ('good' test signal)
- Measure output



Open-loop tuning

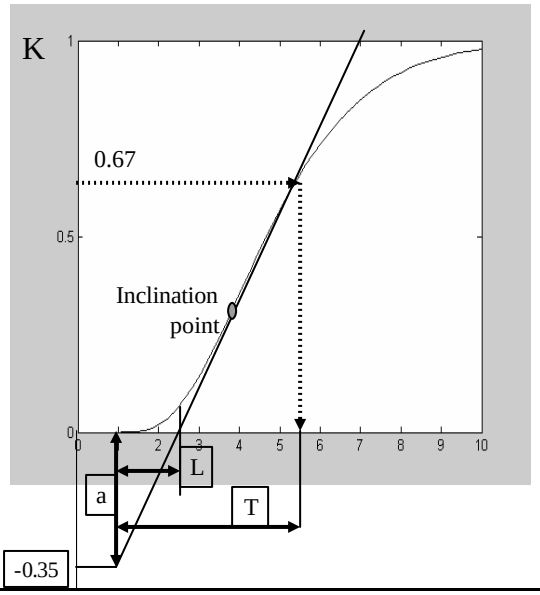




Often in process control unit step response looks like 'S-curve'.

Ziegler-Nichols idea: Approximate the dynamics with a simple transfer function.
The most common: Delay and first order dynamics.

Open-loop tuning – unit step response

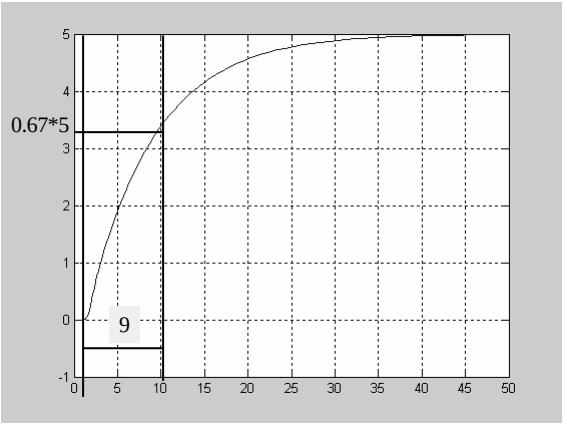


$$G(s) = \frac{K}{1+sT} e^{-sL}$$

From figure:

K=1
L ≈ 1.3 s
T ≈ 4.4 s
a ≈ 0.35

Open-loop Ziegler Nichols

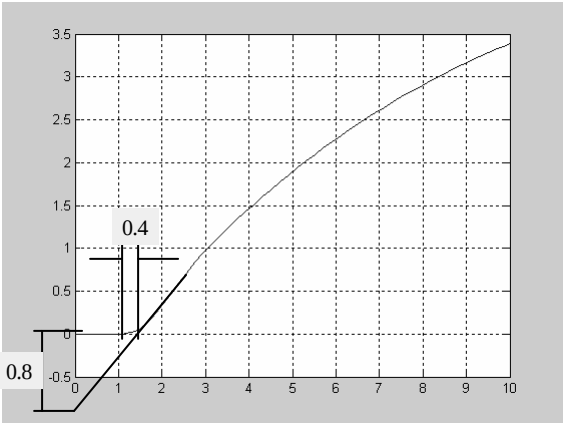


From figure:

K=5

T ≈ 9 s

Open-loop Ziegler Nichols



From figure:

$K=5$
$L \approx 0.4\ s$
$T \approx 9\ s$
$a \approx 0.8$

Open-loop Ziegler Nichols

PID Controller parameters; Ziegler-Nichols step-response method								
Control	K	Ti	Td	Tp = Estimate of				
				oscillation period				
P	1/a			4L				
PI	0.9 a	3L		5.7L				
PID	1.2/a	2L	L/2	3.4L				

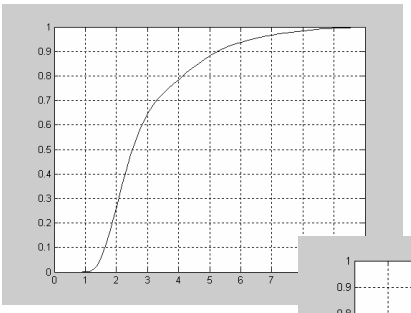
Open-loop Chien, Rhones Reswick

PID Controller parameters;							
Chien, Rhones Reswick load disturbance response method							
		0 %			20 %	overshoot	
Control	K	Ti	Td		K	Ti	Td
P	0.3/a				0.7/a		
PI	0.6 a	4L			0.7 a	2.3L	
PID	0.95/a	2.4L	0.42L		1.2/a	2L	0.42L

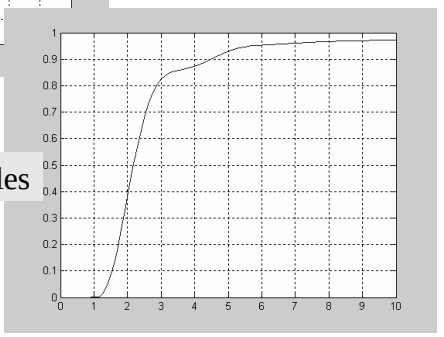
Open-loop Chien, Rhones Reswick

PID Controller parameters;								
Chien, Rhones Reswick setpoint response method								
		0 %			20 %	overshoot		
Control	K	Ti	Td		K	Ti	Td	
P	0.3/a=0.4				0.7/a=0.9			
PI	0.35 a=0.3	1.2T=10.8			0.6 a=0.5	T=9		
PID	0.6/a=0.8	T=9	0.5L=0.2		0.95/a=1.2	1.4T=12.6	0.47L=0.2	

Open-loop Chien, Rhones Reswick

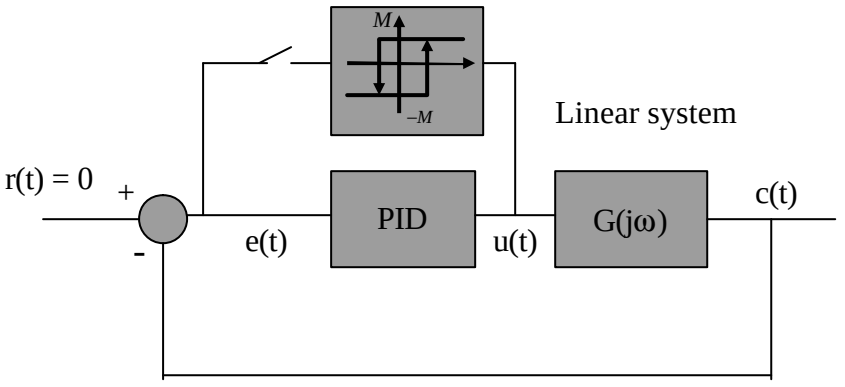


PID, 0% overshoot rules



PID, 20% overshoot rules

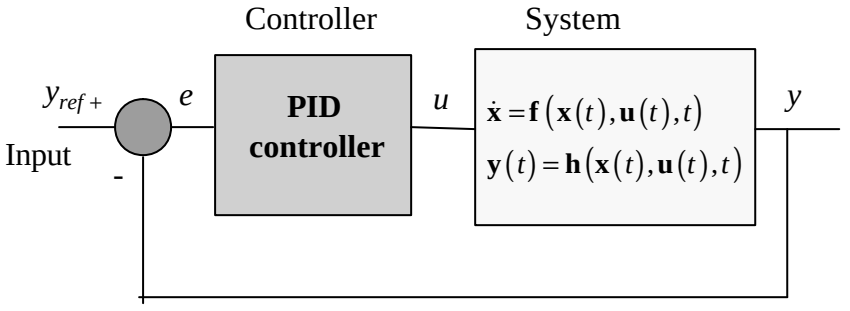
Relay tuning



First relay on, to obtain critical gain K_{cr} and critical period T_{cr}

Optimization of control parameters

Example – Best PID control parameters



PID controller
Parameters K_p, K_i, and K_d

$$u(t) = K_p e + K_i \int_0^t e(a)\,da + K_d \frac{de}{dt}$$

Cost function

- Many cost functions can be used in optimizing PID control parameters.
- Some of the most common ones are

$$ISE = J\left(K_p, K_i, K_d\right) = \int_0^\infty e^2\left(a; K_p, K_i, K_d\right) da$$

$$IAE = J\left(K_p, K_i, K_d\right) = \int_0^\infty \left| e\left(a; K_p, K_i, K_d\right) \right| da$$

$$ITAE = J\left(K_p, K_i, K_d\right) = \int_0^\infty a \left| e\left(a; K_p, K_i, K_d\right) \right| da$$

Parameter estimation methods

System model

$$y(n) = -a_1 y(n-k) - \cdots - a_n y(n-k-n) + b_0 u(n-k) + \cdots + b_m u(n-k-m) + kohina$$

Identify parameters $a_1,...,a_n$ $b_1,...,b_m$

PID controller

$$u(n) = u(n-1) + c_0 e(n) + c_1 e(n-1) + c_2 e(n-2)$$

Determine $c_0,...,c_2$

Rule-based methods

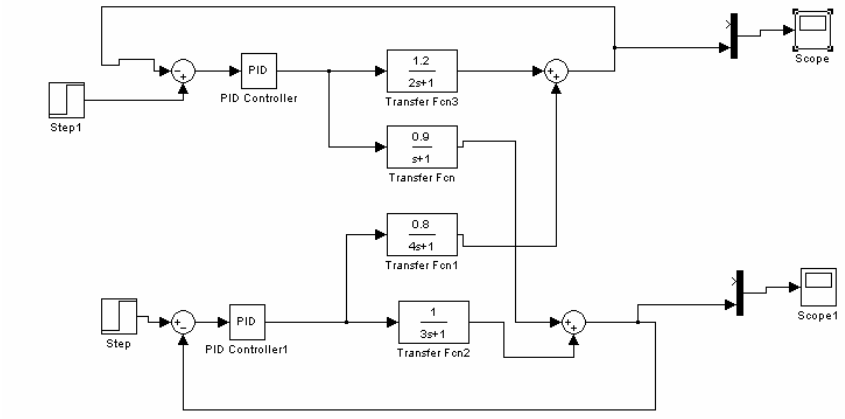
Rules of thumb for the effects of controller parameters

	Speed	Stability
K increases	Increases	Reduces
Ti increases	Reduces	Increases
Td increases	Increases	Increases

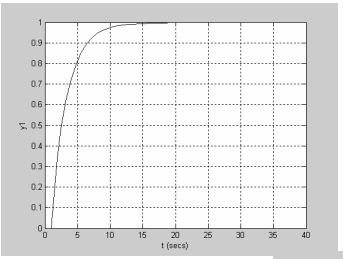
Gain scheduling

Adaptive control

MIMO PID control- Decentralized

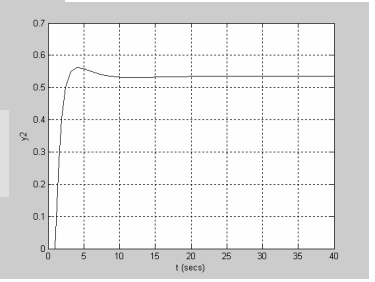


MIMO PID control- Decentralized

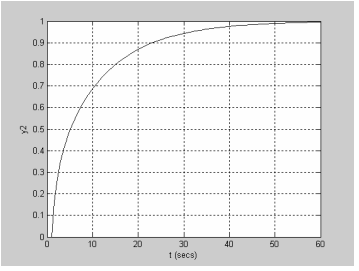


Lieslehto
K=0.83
T_i*=2, I=0.5*
Unit step in ref1

Amount of interaction

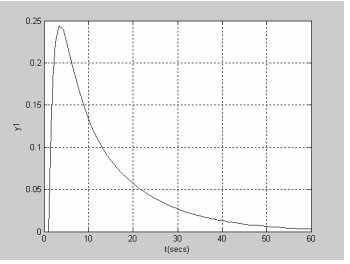


MIMO PID control- Decentralized



Lieslehto
K=1.5
T_i*=2, I=0.5*
Unit step in ref1

Amount of interaction



MIMO PID control - Centralized

State-space equation

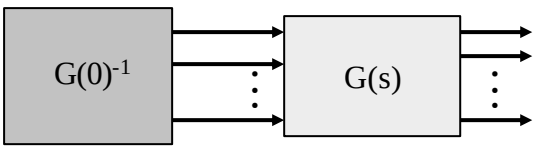
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$
$$\mathbf{y} = \mathbf{C}\mathbf{x}$$

Transfer function

$$\mathbf{G}(\mathbf{s}) = \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}$$

MIMO PID control - Centralized

Decoupling



MIMO PID control - Centralized

$$G(0)=\begin{bmatrix}1.2 & 0.9\\ 0.8 & 1\end{bmatrix}$$

$$K_i = e\left(G(0)\right)^{-1} = e\begin{bmatrix}1.2 & 0.9\\ 0.8 & 1\end{bmatrix}^{-1} = e\begin{bmatrix}2.1 & -1.9\\ -1.7 & 2.5\end{bmatrix}$$

MIMO PID control - Centralized

$$\lim_{s\to 0} sG(s) = \lim_{s\to 0} s\begin{bmatrix}\frac{1.2}{2s+1} & \frac{0.9}{s+1}\\ \frac{0.8}{4s+1} & \frac{1}{3s+1}\end{bmatrix} = \begin{bmatrix}0.6 & 0.9\\ 0.2 & 0.3\end{bmatrix}$$

$$K_p = d\left(\lim_{s\to 0} sG(s)\right)^{-1} = d\begin{bmatrix}0.6 & 0.9\\ 0.2 & 0.33\end{bmatrix}^{-1} = d\begin{bmatrix}18.3 & -50\\ -11 & 33\end{bmatrix}$$