Methods for Tuning PID Controllers

There are several methods for tuning a controller. This paper summarizes several (but not all) of the most common methods. In this paper a PID controller is modeled by the function:

$$p(t) = \bar{p} + K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(t)dt - \tau_D\frac{de(t)}{dt}\right]$$

Trial and error

Assume the controller algorithm is as follows

1. Eliminate the integral and derivative action by setting τ_D to 0 and τ_I to as large a value as possible
2. Set K_c at a low value and put the controller on automatic
3. Increase the controller gain K_c by small increments until continuous cycling occurs after a small set point or load change. The term "continuous cycling" refers to a sustained oscillation with constant amplitude.
4. Reduce K_c by a factor of 2.
5. Decrease τ_I in small increments (this INCREASES integral control) until continuous cycling occurs again. Set τ_I to 3 times this value
6. Increase τ_D until continuous cycling occurs. Set τ_D equal to one third of this value.

Disadvantages

1. It is quite time consuming if a large number of trial are required or if the process dynamics are slow. Testing can be expensive because of lost productivity or poor product quality
2. Continuous cycling may be objectionable because the process is pushed to the stability limit. Consequently, if external disturbances or a change in the process occurs during controller tuning, unstable operation or a hazardous situation could result.
3. The tuning process is not applicable to processes that are open loop unstable because such processes typically are unstable at high and low values of Kc but are stable at an intermediate range of values.
4. Some simple processes do not have an ultimate gain (e.g. first and second order OLTF without time delays)

Note:
If the OLTF is known then K_c can be determined from a root locus plot as the minimum value of K that causes the locus to pass into the right hand side of the s plane.

Continuous cycling method (Ziegler Nichols method)

1. Determine the ultimate controller gain K_{cu} as described in the first three steps, above (or from the root locus plot of the OLTF)
2. The period of oscillation at this value is the ultimate period, P_u. The Ziegler Nichols settings are calculated from these values to provide a ¼ decay ratio

Ziegler Nichols Controller Settings Based on the Continuous Cycling Method

Controller	K_c	τ_I	τ_D

P	0.5 K_{cu}	--	--
PI	0.45 K_{cu}	P_u /1.2	--
PID	0.6 K_{cu}	P_u /2	P_u /8

Adopted from *Process Dynamics and Control* by Dale E. Seborg, Thomas F. Edgar and Duncan A. Mellichamp, Wiley, 1989

Process Reaction Curve Methods (Ziegler-Nichols and Cohen-Coon methods)

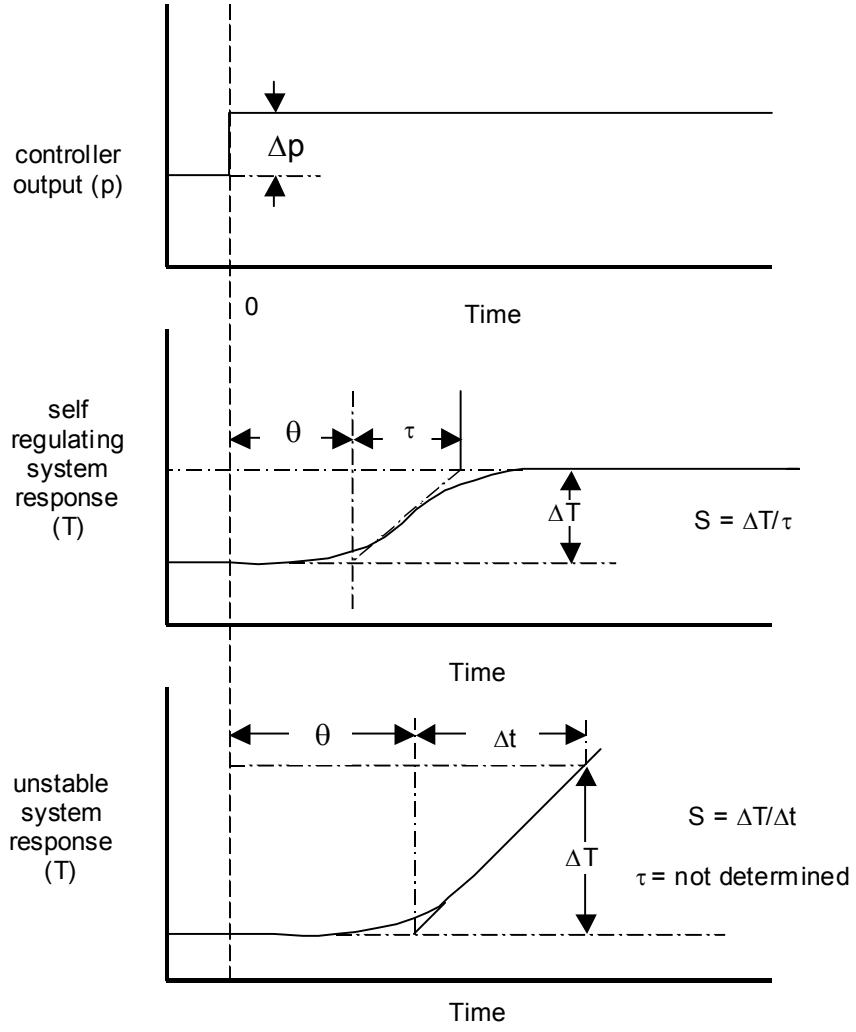
Assume the controller algorithm is as follows

$$p(t) = \bar{p} + K_c\left[e(t) + \frac{1}{\tau_I}\int_0^t e(t)dt - \tau_D\frac{de(t)}{dt}\right]$$

For self-regulating responses (Cohen-Coon method), also assume that the process model can be described as a first order lag with dead time. In other words, the OLTF can be approximated by the following transfer function.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s + 1}$$

1. With the controller in the manual mode, the controller output is suddenly changed. The process output curve follows one of the curves shown in shown below and the times are noted. In the second curve, the straight line to determine τ and the slope S is drawn tangent to the system response at the inflection point. In the bottom curve, the slope S is taken from a straight portion of the ascent



2. **Ziegler-Nichols method (both types of responses)**
   Compute the following:
   $S = \Delta T/\tau$ or $S = \Delta T/\Delta t$ (the average slope of the rise)
   $S^* = S/\Delta p$ (the normalized slope)

3. Compute the controller settings from the following table

Ziegler-Nichols Tuning Relations (Process Reaction Curve Method)

Controller Type	K_c	τ_I	τ_D
P	$\frac{1}{\theta S^*}$	--	--
PI	$\frac{0.9}{\theta S^*}$	3.33θ	--
PID	$\frac{1.2}{\theta S^*}$	2θ	0.5θ

1. **Cohen-Coon method (self regulating response ONLY)**
   Alternatively, the constants K, τ and θ can be used in the process model when the system exhibits self-regulating response (i.e. the response curve follows the second curve so τ can be determined). Controller settings can be calculated using the Cohen-Coon relations (designed for ¼ decay ratio).

Cohen and Coon Controller Design Relations

Controller	Settings	Cohen-Coon
P	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[1+\frac{\theta}{3\tau}\right]$
PI	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[0.9+\frac{\theta}{12\tau}\right]$
	τ_I	$\frac{\theta[30+3(\theta/\tau)]}{9+20(\theta/\tau)}$
PID	K_c	$\frac{1}{K}\frac{\tau}{\theta}\left[\frac{16\tau+30}{12\tau}\right]$
	τ_I	$\frac{\theta[32+6(\theta/\tau)]}{13+8(\theta/\tau)}$
	τ_D	$\frac{4\theta}{11+2(\theta/\tau)}$

Advantages

1. Only a single experiment is necessary
2. Does not require operating at stability limit
3. Does not require trial and error
4. Controller settings are easily calculated

Disadvantages

1. The experiment is performed under open loop conditions (controller on manual). Thus, if a significant load change occurs during the test, no corrective action is taken and the test results may be easily distorted.
2. It may be difficult to determine the slope at the inflection point accurately, especially if the measurement is noisy and a small recorder chart is used
3. The methods tends to de sensitive to controller calibration errors.
4. The Cohen and Coon recommended settings tend to result in oscillatory responses since they were developed to provide a ¼ decay ratio
5. The method is not recommended for processes that have oscillatory open-loop responses since the process model will be quite inaccurate

Additional Comments on the process model.

The process model used for this tuning method (first order lag plus dead time) is actually much applicable than might seem on first consideration. The explanation for this fact lies in the Pade approximation for the Laplace transform of dead time:

$$e^{-\theta s} \cong \frac{1-\frac{\theta}{2}s}{1+\frac{\theta}{2}s}$$

Substituting this approximation into the OLTF yields

$$G(s) \cong \frac{1-\frac{\theta}{2}s}{\left(1+\frac{\theta}{2}s\right)(\tau s+1)}$$

From this approximation, we can see that the system model approximates a system of two first order lags and a first order lead. Many systems can be fit to this approximate model.

ITAE Criteria

In order to establish values of the settings of a tuner, we must first establish criteria (the previous notes used ¼ decay ration as the criteria. There are several possible criteria that we can use, some are summarized below:

Name of criterion	Formula
Integral of the absolute value of of the error IAE	$IAE = \int_0^\infty \lvert e(t)\rvert dt$
Integral of the squared error ISE	$ISE = \int_0^\infty e(t)^2 dt$
Integral of the time weighted absolute error ITAE	$ITAE = \int_0^\infty t\lvert e(t)\rvert dt$

Of these ITAE is preferred because it results in conservative settings. ITAE is conservative because it p

Process and Controller Models

In addition to the criteria for the response, we can also obtain different responses depending on what is modified (load or set point). To mathematically determine the controller setting a process model will be chosen. The first order lag with dead time is a general model that describes the behavior of many processes. Note that a conventional first order lag is a specialized case of this more general model in which $\theta = 0$.

$$G(s) = \frac{Ke^{-\theta s}}{\tau s+1}$$

The controller model is

$$G_c(t) = K_c\left[e(t) + \frac{1}{\tau_I}\int_0^{-\infty} e(t)dt - \tau_D\frac{de(t)}{dt}\right]$$

ITAE Design Formulae and Example

With the criteria, process model, and controller model established, it is possible to mathematically determine the controller settings for a FOPDT process. Note that the ITAE for a P controller is infinity, so we cannot use this method on P only controllers.

Controller Design Relations Based on ITAE Performance Index and First Order plus Time Delay Model

Type of Input	Type of	Mode	Formula	A	B

	Controller				
Load	PI	P	$KK_c = A(\theta/\tau)^B$	0.859	-0.977
		I	$\tau/\tau_I = A(\theta/\tau)^B$	0.674	-0.680
	PID	P	$KK_c = A(\theta/\tau)^B$	1.357	-0.947
		I	$\tau/\tau_I = A(\theta/\tau)^B$	0.842	-0.732
		D	$\tau_D/\tau = A(\theta/\tau)^B$	0.381	0.995
Set Point	PI	P	$KK_c = A(\theta/\tau)^B$	0.586	-0.916
		I	$\tau/\tau_I = A + B(\theta/\tau)$	1.03	-0.165
Set Point	PID	P	$KK_c = A(\theta/\tau)^B$	0.965	-0.85
		I	$\tau/\tau_I = A + B(\theta/\tau)$	0.796	-0.1465
		D	$\tau_D/\tau = A(\theta/\tau)^B$	0.308	0.929

Example:

Use the ITAE (set point) method to estimate PID controller settings for a process with the transfer function

$$G(s) = \frac{5e^{-2s}}{3s+1}$$

Solution:

Determine K

$KK_c = A(\theta/\tau)^B$

$K_c = (A/K)(\theta/\tau)^B$

$K_c = (0.965/5)(2/3)^{-0.85}$

$K_c = 0.272$

Determine τ_I

$\tau/\tau_I = A + B(\theta/\tau)$

$\tau_I = \tau/[A + B(\theta/\tau)]$

$\tau_I = 3/[0.796 - 0.1465(2/3)]$

$\tau_I = 4.296$

Determine τ_D

$\tau_D/\tau = A(\theta/\tau)^B$

$\tau_D = \tau A(\theta/\tau)^B$

$\tau_D = 3 \times 0.309(2/3)^{0.929}$

$\tau_D = 0.636$

adapted from Seborg, D.E., Edgar, T.F., and Mellichamp, D.A. *Process Dynamics and Control*, Wiley, New York, 1989